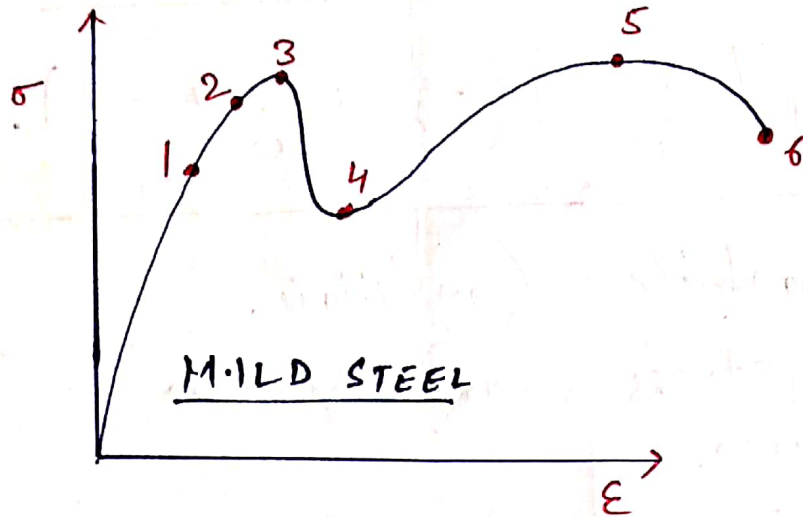


Stress Strain Curve



1 = PROPORTIONAL LIMIT

2 = Elastic Limit

3 = Upper yield point

4 = Lower yield point

5 = Ultimate tensile strength

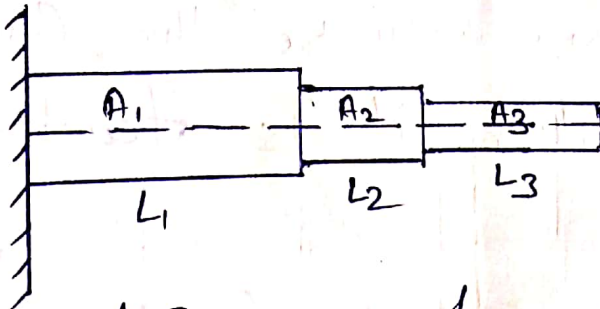
6 = Failure

LOADS IN BARS - SERIES & Parallel

① Bars in Series Conditions

- ① Bar should be prismatic.
- ② Pure Axial loading.
- ③ Bar is made up of same material.

Case I



A Bar is shown having three different

(22)

Areas Component. A load P is applied at the end of the bar. So

Elongation $\delta_{Total} = \delta_{Bar1} + \delta_{Bar2} + \delta_{Bar3}$

$$\delta_{Total} = \frac{P L_3}{A_3 E_3} + \frac{P L_2}{A_2 E_2} + \frac{P L_1}{A_1 E_1}$$

$$\delta_{Total} = P \sum_{i=1}^n \left[\frac{L_i}{A_i E_i} \right] \left[\text{Generalised formula for Total deformation} \right]$$

Now

$\sigma_{maximum}$ = Will be at the section that have minimum Area.

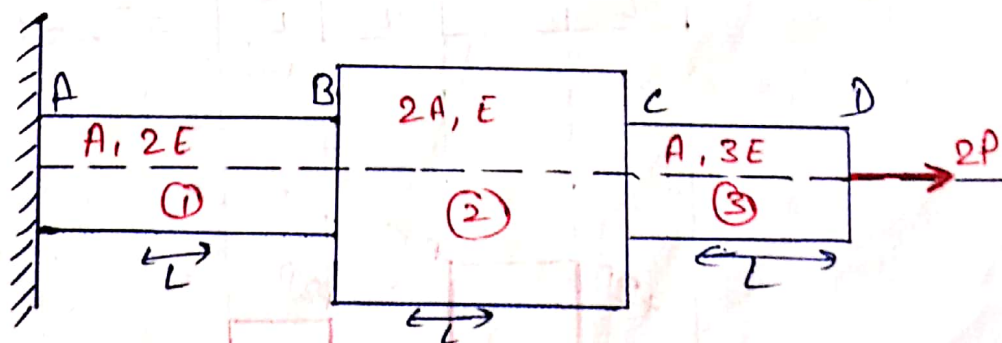
$$\sigma_{maximum} = \frac{P}{A_3}$$

$$\sigma_{minimum} = \frac{P}{A_1}$$

So $\frac{\sigma_{max}}{\sigma_{min}} = \frac{A_1}{A_3} \Rightarrow \left[\frac{d_1}{d_3} \right]^2$

Question For the stepped bar as shown in figure. Determine (i) $\sigma_{max}/\sigma_{min}$ (ii) Deformation at free end if deformation at B point is δ .

Sol



Now (i) $\sigma_{max}/\sigma_{min} \Rightarrow \sigma_{max} = \frac{2P}{A}$ & $\sigma_{min} = \frac{2P}{2A}$

So

$$\frac{\sigma_{\max}}{\sigma_{\min}} = \frac{2P \times 2A}{A \times P} = (2)$$

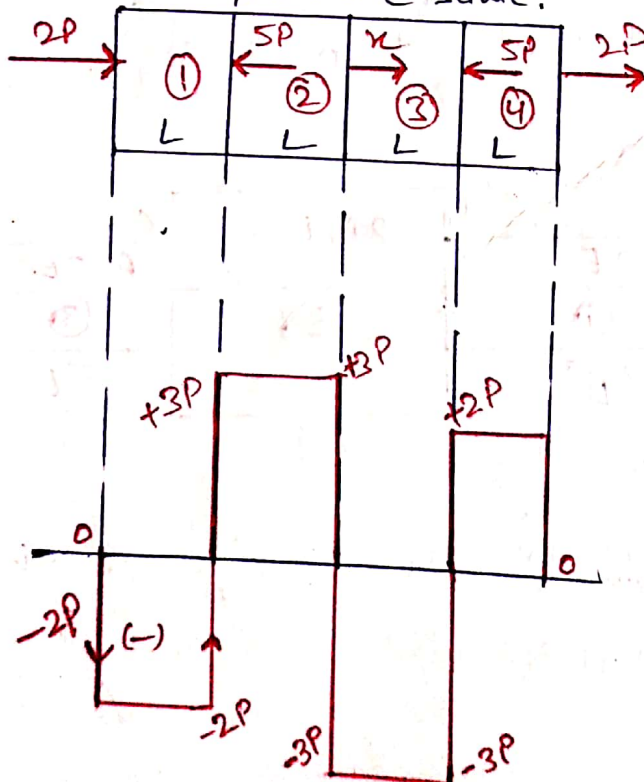
Now To find the deformation at any point in the bar always take fix point as reference point & deformation is with fix point.

$$\begin{aligned} \delta_{\text{Total}} &= \delta_1 + \delta_2 + \delta_3 \\ &= \frac{2PL}{A \times E} + \frac{2PL}{2AE} + \frac{2PL}{3EA} \\ &= \delta + \delta + \frac{2}{3} \frac{PL}{EA} \\ &= \delta + \delta + \frac{2}{3} \delta \Rightarrow \frac{8\delta}{3} \end{aligned}$$

Now

Case II For the bar shown Determine max stress induced, Ratio of $\sigma_{\max}/\sigma_{\min}$ & Deformations.

A & E are same.



Now put load linearly
 $-2P + 5P - X + 5P - 2P = 0$
 $X = 6P$

(23)

Now Obtain from the diagram.

$$P_1 = -2P \text{ [compressive]}$$

$$P_2 = +3P \text{ [tensile]}$$

$$P_3 = -3P \text{ [compressive]}$$

$$P_4 = +2P \text{ [tensile]}$$

So $\sigma_{\max}$ = It will be depend on Areas of bar
But Areas are equal so Intensity of Loads. i.e.

$\sigma_{\max}$ at P_2 & P_3 so
or

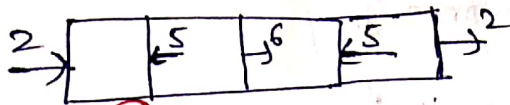
$$\sigma_{\max} = \frac{3P}{A} \text{ \& \text{ similarly } } \sigma_{\min} = \frac{2P}{A}$$

$$\text{So } \frac{\sigma_{\max}}{\sigma_{\min}} = \frac{3}{2}$$

Now Elongation.

$$\begin{aligned} \delta_{\text{Total}} &= \delta_1 + \delta_2 + \delta_3 \\ &= \frac{L}{AE} [P_1 + P_2 + P_3] = \text{Zero} \end{aligned}$$

Another method of solving load.

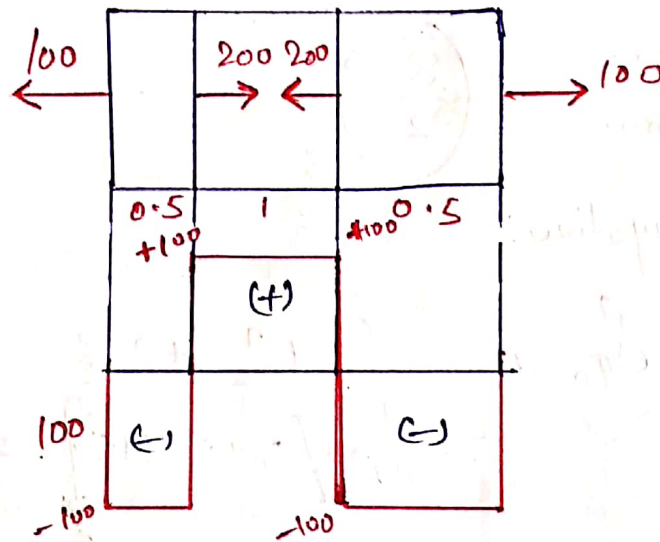


$$\begin{aligned} \text{So } P_1 &= -2 \\ P_2 &= -2 + 5 = 3 \\ P_3 &= -2 + 5 - 6 = -3 \\ P_4 &= -2 + 5 - 6 + 5 = 2 \end{aligned}$$

- * Individual members have non-zero deformations, But total deformation is zero.
- * Load acting on any member = Load acting at non-junction point of the member.
- * Junction load is divided into members hence not real load.

Question A cylindrical Bar of Area = 100 mm^2 is subjected to loading & $E = 200 \text{ GPa}$. Find $\Delta L = ?$

Sol.



So

$$P_1 = -100 \text{ [compressive]}$$

$$P_2 = +100 \text{ [Tensile]}$$

$$P_3 = -100 \text{ [compressive].}$$

So

$$\Delta_{\text{total}} = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE}$$

$$\Delta_{\text{total}} = \frac{1}{AE} [P_1 L_1 + P_2 L_2 + P_3 L_3].$$

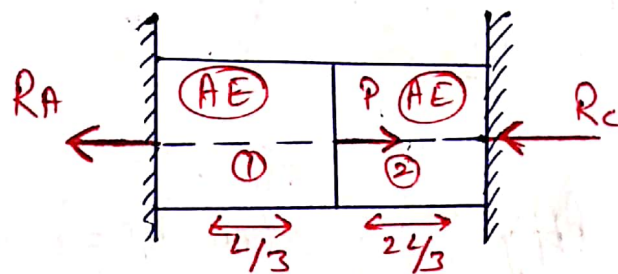
$$\delta_T = \frac{1}{AE} [-100 \times 0.5 + 100 \times 1 - 100 \times 0.5]$$

$$\delta_T = \frac{1}{AE} [0] \Rightarrow \boxed{\delta_T = 0}$$

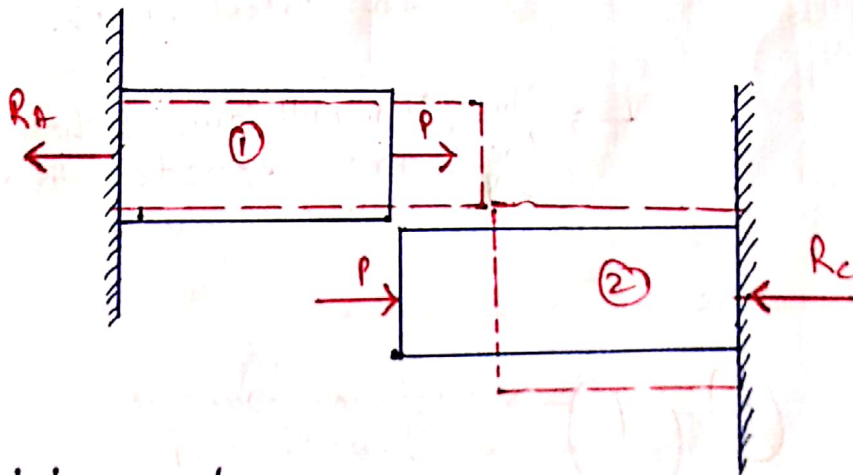
Case III Bar Fixed at both ends (Statically indeterminate Bar.)

$\left| \begin{array}{l} \text{No. of Ryn in} \\ \text{the Bar} \end{array} \right| > \left| \begin{array}{l} \text{No. of Useful Static} \\ \text{Equilibrium equation} \\ \text{ie } \sum V = \sum H = \sum M = 0 \end{array} \right|$

Solution



Initially take rxn at ends at the opposite directions of acting loads.



Taking algebraic sum $R_A - P + R_C = 0$
 $R_A + R_C = P$

&

$\delta_1 + \delta_2$ will be equal to zero.

So

$$\frac{P_1 L_1}{AE} = -\frac{P_2 L_2}{AE}$$

$$\Rightarrow P_1 L_1 = -P_2 L_2$$

$$R_A L_1 = -(R_A - P) L_2 \quad [P = R_A \text{ opposite}]$$

$$R_A \frac{L}{3} = -(R_A - P) \frac{2L}{3}$$

$$R_A = -2R_A + 2P$$

$$R_A = \frac{2P}{3}$$

$$\text{So } R_C = \frac{P}{3}$$

Second Method Applied only when $AE = \text{Constant}$ for all.

$$R_{yn} \text{ at fix end} = \left[\frac{\sum \left[\text{Load acting} \times \text{Distance from the opposite end} \right]}{\text{Total length}} \right]$$

Plus sign will come when R_{yn} & Load are opposite direction otherwise -ve.

For the above case:-

$$R_A = \frac{P \times \frac{2L}{3}}{L} \quad \begin{array}{l} \text{Distance from the} \\ \text{opposite end} \\ \text{Total length.} \end{array}$$

Only one Load is acting

$$R_A = \frac{2P}{3} = P_1$$